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"You're Not Allowed to Give Us the Solution, but Can You Guide Us towards It?"

Insights into Adaptive Teaching Interventions through a Study of Mathematics Teachers

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Abstract: In task-based mathematics classes, where several solution approaches may be valid, diagnosing students' solution processes and adaptively supporting them is a significant challenge for teachers. The question arises as to how mathematics teachers can successfully support students in solving these mathematical tasks and which aspects constitute adaptive teaching interventions. However, few studies have analyzed adaptive teaching interventions in the context of mathematics didactics. This paper first presents a detailed model of adaptive teaching interventions. Based on this model, the interventions of four secondary school mathematics teachers were analyzed using two methodological approaches. The applicability of the model was analyzed, and factors that hinder the adaptivity of teachers' interventions were identified. The results show that 41–63 percent of all identified teaching interventions were adaptive. Up to 55 percent of the teachers' interventions took place even though the students were not facing barriers within their learning processes. The sampled teachers faced major difficulties in diagnosing and in addressing the barriers in students' solution processes. Furthermore, up to 65 percent of all teaching interventions analyzed did not enable students to overcome barriers in their solution processes.

Keywords: adaptive teaching interventions; adaptivity; mathematical problem solving; student-teacher interactions

1 Introduction

Based on the idea that teachers adapt their interventions to their students' needs (Webb, 1991), the construct of adaptive teaching is highly important for determining the effectiveness of teachers' interventions (Dann et al., 1999) and the successfulness of students' learning processes. Adaptive teaching not only helps learners achieve their goals more effectively, but also fosters a more personalized learning environment tailored to the individual needs and abilities of each learner, which can make learning more effective. By adapting to individual needs and abilities, teachers can improve educational opportunities by providing targeted support and an individualized learning environment for learners with different abilities and backgrounds (D'Mello & Graesser, 2012; VanLehn, 2011). To provide individualized support and teach adaptively, teachers need to have macro level competencies, such as knowledge of the topic they are teaching and the capacity to differentiate according to the students' ability level. Furthermore, teachers must be able to intervene in learning situations by diagnosing and reacting to them on a micro level (cf. Lemmrich et al. in the introductory chapter of this special issue). While previous research has provided insights into teachers' competencies on the macro (e.g., that teachers' knowledge is an important predictor of student learning; Hattie, 2009) and micro levels, little is known about the interdependency between the two levels. This gap in the literature might exist because of the complexity of these competencies, which complicates their elicitation. Moreover, researching micro-level competencies must be more subject-specific because of their close interconnections to specific subject matter.

Only a few studies on mathematics didactics have analyzed adaptive teaching interventions. Existing research shows that diagnosing students' solution processes and adaptively supporting them is extremely challenging for future teachers. Furthermore, the more heterogeneous the solution processes that teachers must deal with (e.g., owing to the existence of multiple valid solutions to a task), the more they struggle to intervene adaptively (Cooper, 2009; Dann et al., 1999; Leiss, 2007; Meloth & Deering, 1999; Seifried & Wuttke, 2010; Tropper et al., 2015). To counteract this, teaching interventions need to be analyzed with respect to what makes them adaptive or non-adaptive. Building on this, approaches to facilitate adaptive teaching interventions and thereby improve teaching interventions may be developed. Although previous studies on the didactics of mathematics have analyzed teaching interventions, most do not focus on interventions' adaptivity. This lack of research is aggravated by the fact that no normative model for measuring interventions' adaptivity has been proposed.

This paper presents the current state of research on adaptive teaching interventions from the perspective of mathematics didactics, including a process model for teaching interventions. Based on the research findings, this paper introduces a model developed by the authors to measure the adaptivity of teaching interventions. Subsequently, the paper considers the model's implications for teaching interventions. The interventions of four experienced secondary school mathematics teachers were analyzed according to this model.[1] The analysis used data from two explorative studies conducted using different methodological approaches: video-based lesson observations and a computer-based test instrument. These studies, which were chosen to facilitate an integrated analysis of the model's applicability, are described, and their results are presented. Challenges faced by teachers regarding their interventions' adaptiveness, such as addressing barriers to solution processes, were identified based on the results of the two studies. Moreover, the

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extent to which the two methodological approaches are suitable for measuring the same construct of teaching intervention adaptivity is discussed. Finally, implications for further scientific research are presented.

2 Theoretical background

This section defines the constructs of teaching interventions. Furthermore, a process model of teaching interventions proposed by Leiss (2007) is presented to better understand the teacher's role within the intervention process (see Sect. 2.1). In Section 2.2, the current state of research measuring teaching intervention effectiveness is described. Then, in Section 2.3, a model developed by the authors to measure the adaptivity of teaching interventions is presented.

2.1 Teaching interventions

Teaching is an interrelation between student activity, teacher activity, and student-teacher interactions (Jürgens & Rolff, 2010). In addition to providing learning input, one of the teachers' main tasks in all subjects is to facilitate and support students' learning processes. Every strategic input that takes place in student-teacher interactions during a student's learning process, which aims to facilitate or monitor this learning process, can be defined as a teaching intervention (e.g., Fürst, 1999; Kiper & Mischke, 2009; Scherres, 2013). The goal of each teaching intervention should be to encourage students to work independently and constructively, activate their prior knowledge, and supplement missing information or strategies in mathematics. According to Wood et al. (1976), this support, by which an expert enables learners to "achieve a goal which would be beyond his unassisted efforts" (Wood et al., 1976, p. 90), is a form of scaffolding. Serrano (1996) describes support within a teacher-student interaction via the tasks of demonstrating, giving hints, and promoting students' own thinking and assessments. Leiss (2007) developed a process model to describe interventions as part of the mathematics teaching process (see Fig. 1).

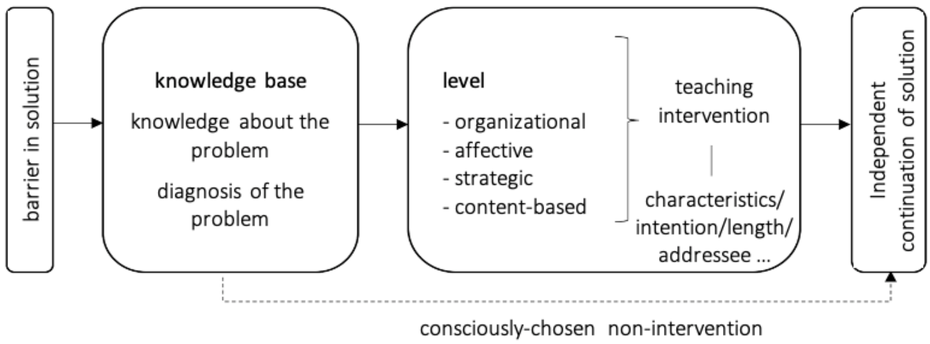

Figure 1: Process model of teaching intervention by Leiss (2007, p. 82) (author's own translation)

Students encounter barriers at certain points in their solution process, which is where a teaching intervention starts. Intervening requires understanding and being able to diagnose the barriers students face. Teachers must consider the potential barriers that students face, their knowledge of the students, the situation, and the learning process as a whole.

Based on this knowledge, teachers may choose to actively intervene in students' learning processes. If teachers decide to intervene actively, they need, in addition to established knowledge, a repertoire of interventions to support students as adaptively as possible. Teaching interventions may include organizational or motivational support. Furthermore, an intervention can support the content or strategic approach (or lack thereof). Consequently, interventions can occur at different organizational, affective, content, and strategic levels.[2]

These levels can be understood as the classification of interventions. The verbal intervention “You’ve already accomplished quite a bit of this lesson,” for example, can be classified as an affective intervention. The goal of a teaching intervention is to allow students to pursue their solutions independently (Leiss, 2007).

2.2 Effectiveness of teaching interventions

Several studies have shown that many teaching interventions do not ideally support students in their solution processes (i.e., they are not effective in their goal of supporting learning) (Dann et al., 1999; Ding et al., 2007; Klock & Siller, 2019; Meloth & Deering, 1999; Seifried & Wuttke, 2010; Stender, 2016; Tropper et al., 2015). Therefore, based on Leiss’s (2007) process model of teaching intervention, the question arises of which characteristics make an effective intervention. Previous studies have used different constructs to analyze and describe the effectiveness of teaching interventions. This section summarizes the state of research on intervention effectiveness.

Tropper et al. (2015) address the question of what constitutes adaptivity. They analyzed the teaching interventions in mathematical modelling lessons. They identified three criteria for adaptive interventions. First, they emphasize diagnosis as the basis of interventions. Teachers’ support was based on their diagnosis of student barriers. Furthermore, the support allows students to continue to work independently. Hence, the second criterion was independent follow-up. The third criterion describes the need for an intervention where support is provided in situations in which students would otherwise not be able to continue working independently. In addition, Tropper et al. (2015) demonstrated that few interventions used by five exemplary practicing secondary school mathematics teachers in Germany were adaptive.

To describe effective interventions, van de Pol et al. (2010) refer to the term “scaffolding” and elaborate on its three characteristics. The first characteristic is that the teacher’s support must be adapted to be at or slightly above the student’s proficiency level. Accordingly, this characteristic is associated with the diagnostic criteria described by Tropper et al. (2015). Van de Pol et al. (2010) described this as a contingency. Another characteristic is that teachers gradually withdraw their level and extent of support over time. The timing or pace of this withdrawal depends on the student’s stage of development and competence. This process is referred to as “fading” and is strongly related to the third characteristic, which is the transfer of responsibility. As the teacher withdraws, responsibility for the learning/solution process must be gradually returned to the student.

In the introductory chapter of this special issue, Lemmrich et al. describe an adaptive learning support model. They distinguish between macro and micro level support. According to Lemmrich et al., the macro level comprises an analysis of (a) the learning objective, (b) students’ competencies, and (c) didactical and methodological settings. Thus, the macro level describes the steps necessary in the context of lesson planning. In contrast, the micro level focuses on teachers’ situational actions in the classroom and includes, among other things, the quality of teacher interactions and their didactical precision (Lemmrich et al., pp. 6–23 in this special issue).

[2] Whereas interventions on a content level include teachers’ statements and actions regarding the mathematical task the students are dealing with, strategic teaching interventions describe support that aims at promoting the students’ solution process from a strategic perspective. An intervention on an organizational level comprises support that aims at improving the organizational conditions, e.g., establishing or maintaining a productive working atmosphere. An affective intervention aims at supporting the students’ solution processes by enhancing and sustaining their motivation (Leiss, 2007).

Ding et al. (2007, p. 166) evaluate the effectiveness of interventions using the following three criteria: "Teachers' guidance focusing on the learning goal and students' cognitive obstacles", "Promoting student thinking", and "Encouraging high-level peer discussion". Their results showed that the length, frequency, and selection of teacher interventions influenced teacher quality. In addition, their findings showed that teachers use peer resources to support students' mathematical thinking. In particular, the focus on student thinking is central, as in other studies (e.g., Tropper et al., 2015; van de Pol et al., 2010).

Moreover, Deering and Meloth (1993) first indicated that teacher interventions are often too vague to have an effect. Consequently, they pointed out that teachers should focus on cognitive and metacognitive aspects when providing help to students (Meloth & Deering, 1999). Regarding the effectiveness of interventions, Vygotsky (1978) emphasizes that individual development occurs through support at the relevant zone of proximal development, which is the individual's zone of maximum development in a given context. The aim of Vygotzky's research was to promote the individual competency-based development of students through assistance for the zone of proximal development to be as high as possible.

Dann et al. (1999) examined the interventions of ten teachers in a group-work setting. They found that many situational interventions tend to negatively affect the content development of group discussions. Based on this, Diegritz et al. (1999) advocate that teachers should "intervene as little as possible (i.e., rarely, briefly, preferably not at all)" during group work (Diegritz et al., 1999, p. 346; authors' own translation). In addition to this, Webb et al. (2002) addressed the length of support for group interactions. They argued that after providing the necessary assistance, the teacher should leave the group to provide students with the opportunity to continue working based on the assistance they received. According to Webb et al. (2002), the teacher could and should return to the group in the solution process and assess whether and how the students used the assistance. Aebli (1998) describes the "principle of minimal help" in the context of problem-solving instruction as the basic principle for teachers' support of their students, where the teacher allows students to think independently and only provides minimal support if necessary.

Dann et al. (1999) showed that teachers often behave in a less situation-sensitive manner by going to groups individually and intervening briskly without taking time for a detailed assessment of the situation. This observation has also been reported by other studies (Leiss, 2007; Meloth & Deering, 1999). Seifried and Wuttke (2010) examined teachers' interventions regarding the diagnoses made by teachers and their level of elaboration. In approximately two-thirds of the interventions in their small-sample study, the teacher did not correctly or sufficiently diagnose students' errors. Moreover, most of these diagnoses were followed by interventions with a low degree of elaboration.

In summary, previous research shows that a correct and detailed diagnosis of the situation is the basis for an effective teaching intervention (e.g., Tropper et al., 2015). Furthermore, various characteristics that concretize an adaptive (or effective) intervention and its further course, such as the principle of minimal help or the existence of a need, have been elaborated (e.g., Aebli 1998; Tropper et al., 2015). However, hardly any research has analyzed the adaptivity of real-life teaching interventions. This type of research is necessary to investigate the current state of research on teaching interventions, how adaptive they are, and what causes them to be deficient.

2.3 A model of adaptive teaching interventions

To synthesize the results in the literature, we developed a model to describe an adaptive teaching intervention (see Fig. 2).[3] While the process model by Leiss (2007) presented in Section 2.1 describes the different steps that take place during a teaching intervention, this newly developed model only focuses on those aspects within a teaching intervention that contribute to its adaptivity. For an intervention to be considered adaptive, all five criteria must be satisfied. However, the criteria are not fully built on each other. Criteria 3 and 4 can only be fulfilled when a barrier exists (Criterion 1). All the other criteria were separated. Thus, for example, an intervention can help students overcome their barriers (fulfillment of Criterion 5), even though it does not enable the students to independently continue their solution processes (non-fulfillment of Criterion 4). This section details the five criteria, their meanings, and their connections to the literature.

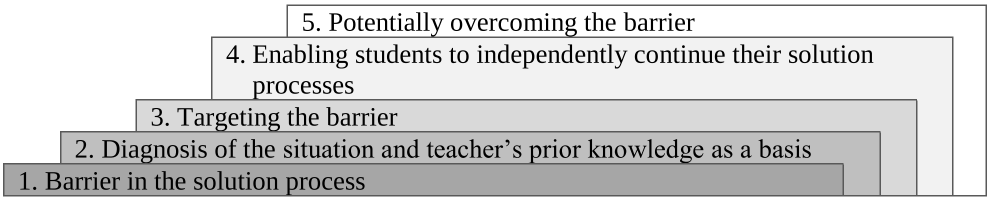

Figure 2: Model of adaptive teaching intervention (own research)

Criterion 1: Barrier in the solution process

It can be assumed that learners face different barriers in their learning processes and need support (Leiss, 2007) that they may or may not be able to express. Regardless of whether learners recognize a barrier, the teacher may perceive a potential barrier that has already led to student problems or could lead to problems. Nevertheless, it should be ensured that teaching intervention in this particular situation is necessary for students to continue their solution processes independently. Thus, a teaching intervention should not disturb the students' learning processes (Diegritz et al., 1999; Tropper et al., 2015). The existence of an insurmountable barrier and, consequently, the need for intervention are the basis of the first criterion.

Criterion 2: Diagnosis of the situation and teacher's prior knowledge as a basis

The first criterion directly leads to the second, as the teacher must diagnose the students' learning processes to assess their needs (Klock & Siller, 2019; Leiss, 2007; Leiss & Wiegand, 2005; Leuders et al., 2018; Tropper et al., 2015; van de Pol & Elbers, 2013; Vogt & Brühwiler, 2020). The teacher has the challenging task of deeply understanding the current state of a student's learning process. Therefore, the following questions must be answered by each student within a relatively short amount of time:

- What is their current state in the solution process?
- What have they already been working on?
- Which approach did they choose?
- Which difficulties do they have to encounter?
- Which mistakes have they already made?

In doing so, the teacher must understand the students' prior knowledge, experiences, strengths, and weaknesses. Furthermore, the teacher needs expertise and knowledge regarding the subject matter to be able to answer these questions for themselves.

[3] A detailed description of the model's derivation will be described in one of the author's doctoral projects (Scharnberg, in preparation).

Criterion 3: Targeting the barrier

Based on the diagnosis in Criterion 2, the teacher selects an intervention that considers the diagnosis of the situation and knowledge about the student. The chosen intervention needs to target barriers in the students' solution process. Intervention can take place at a level different from the barrier itself (Klock & Siller, 2019; Leiss, 2007), but this still needs to be addressed. When students cannot find a solution, for example, and consequently face a content-based barrier, the teacher can intervene at the organizational level by encouraging them to work collaboratively.

Criterion 4: Allowing students to independently continue their solution processes

According to Leiss (2007), the goal of an intervention is to allow students to continue their solution processes independently and as quickly as possible. Students should be guided by individuals with minimal intervention to overcome barriers (Aebli, 1998; Klock & Siller, 2019; Wood et al., 1976). Therefore, a teaching intervention is a balancing act of intervening as much as necessary but as little as possible. The responsibility for the solution process should be transferred back to the student as soon as possible (van de Pol et al., 2010) so that, in the spirit of Vygotsky's (1978) zones of proximal development, students are able to maximize their skill development.

Criterion 5: Potentially overcoming the barrier

Finally, adaptive interventions enable students to overcome barriers (Hermkes et al., 2018; Tropper et al., 2015). Such intervention is ineffective if a student continues to work independently, but does not make progress. Accordingly, in terms of adaptivity, teachers must ensure that students can overcome barriers and continue their learning processes.

2.4 Measurement of teaching interventions

Different measurement methods have been developed in previous research to investigate teaching interventions. To gather data on teaching interventions in a structured, methodically controlled manner, teaching interventions and their effects can be analyzed through lesson observation. Therefore, data on teaching interventions can be collected and analyzed systematically at the moment a teaching intervention takes place within the teaching setting (König, 1973). According to Friedrichs (1990), observations can occur in real-life classrooms or laboratory settings. Observing interventions in real-life teaching situations provides a more realistic picture of the complexity of teacher interventions (Pauli & Reusser, 2006). Since video-based lesson observations enable data collection in real-life classroom settings without the need for analysis, this method is suitable for measuring and analyzing teaching interventions and their adaptivity (Brophy, 2004; Stigler et al., 2000). Through video-based lesson observations, different components of teachers' actions can be measured. Using multiple cameras and microphones allows for recording and analyzing students' actions in detail (Helmke, 2012; Krammer & Reusser, 2005). However, video-based lesson observations have three disadvantages: First, comprehensive video-based lesson observations using multiple camera perspectives and microphones are very time-consuming and expensive with regard to data collection, preparation, and analysis (Hatch & Grossman, 2009; Praetorius, 2014). Second, the use of cameras in the classroom, as well as non-video-based observations, impacts the lesson and may distort the results (Brophy, 2004). Third, observation, whether video-based or not, does not allow the collection of data on teachers' thoughts and knowledge but only on visible or audible information.

Paper-and-pencil or computer-based test instruments can be used to objectively measure teachers' knowledge and diagnostic competencies. The test instrument captures different knowledge and cognitive facets through various items (Klock & Siller, 2019). Beyond the measurement of knowledge, a test instrument enables the collection of data on teachers' decisions on how to intervene in a given student-teacher interaction. In contrast to lesson observation, the situation in a test instrument is artificially created and, therefore, is less realistic. Moreover, it focuses on the cognitive aspects of interventions (Klock & Siller, 2019). Finally, teachers' decisions in a testing situation are made less spontaneously than in a real-life situation.

To analyze the adaptivity of teaching interventions based on the model presented in the previous section and thereby prove the model's suitability for this type of investigation, both video-based lesson observation and a paper-and-pencil or computer-based test instrument are suitable methodological approaches. For the results regarding the model's suitability to be as meaningful as possible and to work out assumptions about which criteria predominantly make teaching interventions non-adaptive, two explorative studies following both methodological approaches were developed and conducted. In the following section, the research questions of this study are presented before detailing the methodologies of both studies in Section 4.

3 Research questions

Based on the current state of research, a model of adaptive teaching intervention comprising five adaptivity criteria was developed (see Sect. 2.3). This study investigated the extent to which experienced math teachers' interventions fulfilled the criteria of adaptive teaching interventions described in the model. This study aimed to analyze the adaptivity of these interventions using these five adaptivity criteria. For an integrated analysis of the model's applicability, this analysis is based on the results of two studies that follow different methodological approaches (see Sect. 4). The model was applied to answer the following two research questions:

1) To what extent do math teachers' teaching interventions, investigated through (a) video-based lesson observations and (b) computer-based test instruments, fulfill the five criteria of adaptive teaching interventions?
2) What can be stated about the adaptivity of math teacher interventions considering (a) the five adaptivity criteria and (b) the two methodological approaches?

4 Methodology

To examine the intervention adaptability of experienced math teachers, two explorative studies using different methodological approaches were conducted between 2018 and 2021. While the first study sought to collect and analyze data on real-life teaching interventions using video-based lesson observations (VLO), the second aimed to capture the diagnostic ability and intervention adaptivity of experienced teachers using a computer-based test instrument (CTI). To answer the research questions, the data from both studies were coded based on the five adaptivity criteria for teaching interventions described previously. The following section presents the samples from both studies. Subsequently, the research methods of both studies are described in detail. Finally, an analysis of the two methodological approaches regarding their suitability for capturing the adaptivity criteria is presented.

4.1 Sample

To improve the comparability of results with respect to the two methodological approaches, both studies used the same sample. As adaptive teaching interventions require deep expertise in the subject matter that is taught, this expertise represents the main criterion for sample selection. To ensure prior knowledge regarding planning, conducting, and analyzing competence-based mathematics lessons, the teachers of the sample have been chosen because of their participation in the "Entwicklungsteam Mathematik"[4] since 2016. Therefore, they were considered experienced, well-practiced math teachers, even though they only had 3.5–11 years of teaching experience with mathematics, and only three thirds of the teachers in the sample had studied mathematics.

Because not all the members of the "Entwicklungsteam Mathematik" have been participating during the conducting periods of both studies, the sample size was small, comprising only four math teachers (one male and three females). As both studies were explorative, the small sample was chosen to be sufficiently representative to work out exemplary assumptions.

4.2 Methodology of Study I – VLO

In the first study, four mathematics lessons of up to 90 minutes each were recorded from up to 13 different camera perspectives.[5] In addition to a tilt camera and clip-on microphone recording the teachers' actions and statements, ten to twelve fixed cameras and microphones were used to record the lessons in detail (see Scharnberg, in preparation).

For comparison, all four teachers used the same mathematical problem to promote their students' mathematical problem-solving competencies. The mathematical problem focused on the connection of a polygon's area and its circumference and consisted of several subtasks with increasing levels of difficulty. It aims to build and expand students' mathematical problem-solving competencies by asking them to find polygons of different shapes with a given circumference and area. To increase the traceability of the solution process, students in all classes were asked to solve the mathematical problem collaboratively in small groups of up to six students and to take notes on their solutions. Therefore, students received a task sheet and teaching materials (matchsticks and square paper) that enabled them to solve the problem symbolically, iconically, and enactively.

To ensure that the teachers had professional knowledge, professional content knowledge, and diagnostic competencies regarding the specific mathematical problems used in the recorded lessons, they were trained before carrying out their lessons. During training, the task space (possible solution approaches and possible barriers within the students' solution processes) was analyzed collaboratively.

Four mathematics classes (one per teacher) were recorded. The length of the collaborative solution processes within the lessons varied between 38 and 50 minutes (see Table 1 on the next page). Consequently, 2,247 minutes of data on collaborative solution processes were recorded and edited, i.e. synchronizing audio and video, anonymizing students within the video and identifying the different phases throughout a lesson. Subsequently, teaching interventions within the collaborative solution processes of each

[4] The "Entwicklungsteam Mathematik" is a team of teachers and researchers at Leuphana University Lüneburg that aims at improving teaching in schools and universities by strengthening the connection between theory and school practice (Ehmke et al., 2021; Scharnberg, 2019). In the context of this collaborative work, the teachers received input on how to plan, conduct and analyze competence-based mathematics lessons. Moreover, they deepened their knowledge by cooperating with university students in the context of a university course. In return, the teachers contributed their experience and knowledge from school practice. Furthermore, they tested collaboratively developed material regarding its suitability for school practice.

[5] Before the recordings, the teachers as well as the students and their parents were informed about the recording and the purpose of the research and provided a written consent.

teacher were identified. In total, 196 teaching interventions were conducted. After identifying all teaching interventions in the dataset, their adaptivity was rated by three independent coders who had been trained before coding. An interrater reliability analysis using the kappa statistic (Landis & Koch, 1977) was performed to determine interrater consistency. Teachers A and C's interventions were coded by one of the authors of this article (Coder 1) and a second coder (Coder 2). The interrater reliability of the two raters was kappa = 0.63. Teachers B and C's interventions were coded by Coder 1 and a third independent coder (Coder 3), respectively. The interrater reliability was kappa = 0.68. Thus, Cohen's kappa indicates a substantial level of agreement between the coders. This outcome suggests that the ratings are generally reliable and consistent, with only a moderate degree of variability. Further analysis revealed discrepancies in the coding, especially regarding the fourth criterion (see Scharnberg, in preparation).

Table 1: Overview of collected data (own research)

Teacher	A	B	D	F
Length of collaborative solution process (mins)	47.00	45.25	38.5	49.75
Total number of teaching interventions	56	51	38	51

The coding scheme (see Table 2) was based on adaptivity Criteria 1, 3, 4, and 5, as described in Section 2.3. The second adaptivity criterion was not measured in this study because it was not possible to measure teachers' pre-existing knowledge and diagnoses by observing their performance within a lesson (see Sect. 2.4). For a teaching intervention to be considered adaptive within this study, all four criteria must be fulfilled.

Table 2: Coding scheme of adaptive teaching interventions (own research)

Adaptivity criterion	*Codes*	*Requirement*	*Example*
There is a barrier within the students' learning processes that they are probably not able to overcome themselves. (C1)	0	The students do not face any barrier within their solution processes immediately before or at the beginning of the teaching intervention.	While the students work on the task, the teacher starts an intervention by asking whether the students need help.
	1	The students face a content-based barrier within their solution processes immediately before or at the beginning of the teaching intervention.	The students are stuck in their solution process and ask the teacher for help. The teacher reacts to their question.
The teaching intervention targets the students' barriers. (C3)	0	The teaching intervention does not target the students' barriers. This might be because (I) there is no barrier within the solution process, (II) the teacher did not diagnose the barrier correctly and is therefore not able to target it or (III) there is a barrier, and the teacher possibly diagnosed it correctly, but the teacher does not talk about the barrier within the intervention.	The students do not understand the task. The teacher thinks they did not work together as a group and reminds them to work as a group.

	1	The teaching intervention targets the students' barriers. The teacher addresses the barrier within the intervention with the objective of supporting students to overcome it.	The students do not work together as a group. The teacher addresses this by reminding them to work together and giving them hints on how to work better as a group.
The teaching intervention enables the students to independently continue their solution processes. (C4)	0	The teacher intervenes or steers strongly so that the students' solution processes are heavily relieved. Hence, the teaching intervention does not enable the students to independently continue their solution processes.	The students do not understand the term area. The teacher explains the term area and visually depicts its meaning by using the materials the students received to solve the problem (paper with squares + matchsticks). After that, the teacher uses the matchsticks to solve the first part of the problem.
	1	The teacher only provides minimal support and thereby enables the students to independently continue their solution processes.	The students do not understand the term area. The teacher explains the term area and visually depicts its meaning by using the materials the students received to solve the problem (paper with squares + matchsticks). After that, the teacher lets the students try to find a solution themselves.
The teaching intervention enables the students to potentially overcome the barriers in their solution processes. (C5)	0	During and after the teaching intervention, the students are not able to overcome the barriers in their solution processes.	When the students ask the teacher for help, the teacher gives a hint that does not help the students to solve the task. The students are still stuck with the same problem after the intervention.
	1	During or after the teaching intervention, the students are potentially able to overcome the barriers in their solution processes.	When the students ask the teacher for help, the teacher gives a hint, which helps the students to solve the task.

To rate C1, students' solution processes that occurred before the beginning of the teaching intervention were analyzed with regard to content-based barriers within the solution processes, such as mistakes or stagnation. The time span of the analyzed solution process varied depending on how long the students had been working on the subtask that was addressed in the teaching intervention. To analyze the students' solution processes, three coders viewed the video recordings of the group tables and reconstructed their solution processes, including their approaches, barriers, and solutions. Moreover, the coders identified the lengths of teaching interventions. To rate C1, the coders checked whether there was an insurmountable barrier in students' solution processes at the beginning of the teaching intervention. Therefore, whether this barrier occurred within a specific timeframe before the start of the intervention is irrelevant. What counts is solely the fact that the students were unable to overcome the barriers themselves until the start of the intervention.

If the barrier identified in the students' solution processes was verbally addressed within the teaching intervention, C3 was fulfilled. Therefore, only the tilted camera recordings of the teaching interventions were analyzed. Teachers can, for example, address a barrier by giving advice and asking about the correctness of a solution ("Are you sure this is correct?") or even by steering students in a different direction without directly intervening ("Think about what we have done in the last lesson").

An adaptive teaching intervention should facilitate students' solution processes without revealing solutions (see Sect. 2.3). To analyze whether the teaching interventions enabled students to continue their solution processes independently, the teachers' actions were recorded using a tilt camera. C4 was fulfilled if the teacher allowed sufficient time and opportunity to process the information after providing hints. If a teacher gave several hints simultaneously without allowing the students sufficient processing time, like in a Socratic dialogue, this criterion was not fulfilled.

To determine whether a teaching intervention enabled students to overcome barriers, the solution processes following teaching interventions were analyzed. Hence, recordings from fixed cameras at the student group tables were used to track the ongoing solution processes. If the students were able to correct their mistakes or continue their solution processes after stagnation, C5 was fulfilled. Even if the students started to overcome a barrier, such as by continuing a solution process, but failed to overcome it because of other issues taking place in the classroom, such as a disruptive student at a group table, this criterion was rated as fulfilled. If a teaching intervention did not enable students to overcome a barrier – for example, because the hint the teacher gave was not helpful –, this criterion was rated as not fulfilled.

Each criterion was scored based on the dichotomous coding of the four adaptivity criteria. If a criterion was fulfilled, it was coded as one, and its score equaled one. If, on the contrary, a criterion was not fulfilled, it was coded as zero, and its score equaled zero. Subsequently, the total score for each intervention was calculated by adding the scores of the four adaptivity criteria. The total score ranged from zero to four. Finally, the total score was transformed into an adaptive score. All interventions with total scores of four achieved an adaptivity score of one and were therefore considered adaptive teaching interventions. All other interventions, regardless of their total scores, achieved an adaptivity score of zero and were consequently considered non-adaptive teaching interventions.

4.3 Methodology of Study II – CTI

In the second study, a computer-based test instrument (CTI) was used to assess teacher diagnosis and intervention competence. The test instrument was designed and piloted as part of a doctoral project (Schilling, in preparation) and was realized via "LimeSurvey®". The test took 90 minutes to complete. Teachers were provided limited time to answer knowledge-based questions to prevent them from researching the internet during the test while also not putting them under time pressure. To ensure that all items were processed as consistently as possible, all test items had to be answered, with no option to skip an item.

The test instrument was divided into three subsections to investigate intervention competence:

In the first subsection, teachers' knowledge of adaptive teacher interventions, as well as didactical knowledge regarding mathematical problem solving, was assessed using six items adapted from Scharnberg and Leiss (2018) (e.g., "Name six heuristic strategies or principles when working on problem-solving tasks"). According to the second adaptivity criterion (see Sect. 2.3), didactical and task-related expert knowledge is necessary for an appropriate diagnosis. These areas were investigated at the beginning of the tests.

In the second subsection, three problem-solving tasks were used to investigate the underlying expertise necessary for the subsequent steps towards diagnosis and intervention. One of them, "Devil" (Kuzle, 2019, p. 42), is shown in Figure 3. Thus, expert knowledge on the selected problem-solving tasks was collected.

> Task "Devil"
> The devil says to a poor man: "Every time you cross this bridge, I will double your money. But every time you come back, you have to throw eight thalers in the water." When the man returned for the third time, he did not have a single thaler left. How many thalers did he have at the beginning?

Figure 3: Example of a problem-solving task (own research)

In the third subsection, the teachers' diagnosis competence and their ability to choose adaptive teaching interventions were measured through various items. Both the diagnostic items and the items for selecting interventions were assessed using a constructed sample case (see Fig. 4) specific to mathematical problem-solving in combination with single- or multiple-choice items. Each sample case is based on the aforementioned problem-solving tasks.

> Sample case
> You are a teacher at a high school, and your 5th grade students are working on the assignment as part of a small project in groups of 3 (S1, S2 and S3). They have about 30 minutes to do this. The students have already gained experience with problem solving tasks. The considered students have an average performance level for the grade level. You observe the learners after 8 minutes of processing time during the conversation shown in the excerpt. You have not intervened in the learning process before. The group has taken notes (see figure).
>
> *S1: "But actually it's quite simple, we just have to go back through the whole thing. That is, he always gets 8 thalers in addition and doesn't throw them away."*
> *S2: "Right, so he has 8 thalers before and then they are doubled."*
> *S1: "Right, so 16 thalers. S3, are you taking notes?"*
> *S3: "Yes, and then 8 thalers are added again, so 24 thalers."*
> *S1: "And that doubled makes 48 thalers."*

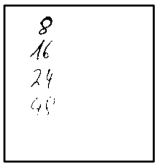

Figure 4: Example of a sample case (own research)

In the first part of the third subsection, the teachers' diagnosis competence was investigated using 21 items (see Fig. 5 on the next page). This process involved analyzing students' solution processes, identifying students' difficulties, and deriving explicit support goals.

Diagnose the learner's barriers in completing the task in this situation.
The learners…
a. … probably have barriers to understanding the task correctly.
b. … probably have barriers to choosing an appropriate solution (heurism).
c. … probably have barriers to applying the heurism correctly.
d. … probably have barriers to reflecting on their solution processes.
e. … probably have no barriers.

Figure 5: Example of a diagnostic item (own research)

As the second part of the third subsection, 30 intervention prompts were rated by each teacher in terms of their suitability regarding the barrier (see Fig. 6). As teachers' suitability ratings for the intervention prompts were based on their assumption that adaptive and suitable interventions describe the same construct, intervention prompts rated as suitable can also be classified as adaptively rated intervention prompts. This study assumed that teachers would carry out an intervention as described in an intervention prompt rated as adaptive. Therefore, for the sake of consistent use of terms within this article, the intervention prompts rated as adaptive will be put on a level with teaching interventions that have been carried out.

Please decide which of the following interventions are suitable in this situation to promote independence-oriented problem-solving skills (suitable vs unsuitable).

a. "Include the mathematical operation in your notation."
b. "What's the task? When do you double and when do you subtract 8 thalers?"
c. "Remember that the man has no thalers left at the end."
d. "When you go back, the devil doesn't double your thalers because he only does that when you go forward. So, you have to cut them in half. That means you have only 8 thalers left and not 16."
e. "I do not intervene (for the time being) and let the students continue working independently."

Figure 6: Examples of intervention prompts (own research)

In total, 24 intervention prompts (four per case) represented active interventions that could be carried out, and six intervention prompts (one per case) represented consciously chosen non-interventions. Owing to the limited time available to complete the test, the cases and items for diagnosis and intervention only focused on content-related barriers.

As part of the analysis, didactical and expert knowledge, teachers' diagnoses, and evaluations of the adaptivity of the interventions were analyzed. To analyze knowledge, all item answers were checked and scored based on their correctness. The scores and total scores determined during the initial implementation of the test instrument were used (Schilling, in preparation). To analyze the interventions regarding their adaptivity, all intervention prompts that were rated as adaptive by the teachers and designed to fulfill the third criterion were scored as one. Moreover, all intervention prompts that were designed not to fulfill the third criterion but were rated as adaptive by the teachers were scored as zero. For example, if a teacher rated intervention prompt d in Figure 6, which targets the barrier, as adaptive, the rating was scored as one for this criterion. The ratings for Criteria 4 and 5 were scored analogously. If deemed adaptive, intervention prompt d in Figure 6 from the previous example aims to overcome the barrier but does not provide the opportunity for the student to continue working independently, so the fourth criterion is scored as zero, and the fifth criterion is scored as one. Because the test instrument only used sample cases in which a barrier was present, the first criterion was fulfilled for all intervention prompts included in the test instrument and, consequently, was always scored as one. To analyze the fulfillment of the second criterion, the total diagnosis score (as described previously) and the total score of subject didactical and expert knowledge were used. Only the diagnosis score was used to assess the criterion score for determining the adaptivity of an intervention. As described in Study I, the scores of the different criteria were added. If the total score was five (i.e., all five adaptivity criteria were fulfilled), the teacher's intervention was considered adaptive. All the interventions that achieved a lower total score were rated as non-adaptive.

4.4 Analysis of the approaches' suitability for capturing the adaptivity criteria

As teachers intervene in their lessons for various reasons, the observation of real-life teaching situations enables the observation of teaching interventions that are not based on barriers to students' solution processes. Observing and analyzing students' solution processes allows for determining whether a teaching intervention is based on student barriers. As the VLO was conducted as a multiperspective observation in which the students' solution processes were recorded in detail, the students' barriers were accessible and visible for data analysis. Therefore, the VLO enabled measurement of the first criterion of adaptive teaching interventions (see Fig. 7). In the construction of CTI cases (Study II), only student solution processes with barriers were designed. This decision was made because of the time-related feasibility of the test instrument. Accordingly, barriers to the solution process were present in all teaching interventions available for selection. Therefore, the first criterion could not be measured using the instruments used in Study II.

Whereas the VLO does not allow the collection of data on whether a teacher might have been able to diagnose a student's barrier, the CTI can be used to adequately assess expert knowledge, especially regarding a specific subject matter (as was done in this study). Moreover, computer-based tests can be used to assess teachers' diagnoses of specific cases in individual steps, either using closed items, as in this study, or open items, thus enabling the measurement of Criterion 2.

Analyzing the content of a teaching intervention taking place in a real-life lesson and relating it to the barriers identified within students' solution processes via VLO allows for determining whether different teaching interventions target students' barriers. To measure this third criterion using a CTI, the intervention options to choose from must be constructed in such a way that some do and others do not target the barrier. This was assessed using computed tomography. Consequently, Criterion 3 can be met using both methodological approaches.

Whether an intervention enables students to independently continue their solution processes instead of giving away the (steps to a) solution can be measured by analyzing the extent to which a teacher intervenes amidst the barriers identified in the students' solution processes via VLO. It is important not only to analyze the length of an intervention but also to consider the content-based aspects the teacher brings up in their intervention. To measure this fourth criterion using the CTI, the test needs to include interventions that enable students to independently continue their solution processes, as well as those that do not. As realized in the CTI, Criterion 4 can be measured using both methodological approaches.

Criterion 5 focuses on whether students can overcome barriers in the solution process. As the multiperspective VLO enables detailed recording and analysis of students' solution processes after a teaching intervention, this aspect can be fully analyzed. In contrast, the CTI can only measure the potential of overcoming the barrier, that is, whether the teacher's action is suitable for the next step in the solution process. This can be achieved by implementing a choice of interventions that potentially enable students to overcome a barrier and those that do not.

Adaptivity Criterion	Can Be Measured by VLO	CTI
1. Barrier in the solution process	✓	(✓)
2. Diagnosis and teacher's prior knowledge	X	✓
3. Targeting the barrier	✓	✓
4. Enabling independent continuation of solution process	✓	✓
5. Potential overcoming of the barrier	✓	✓–

Figure 7: Overview of criteria measurements by measuring method (own research)

Section 5.2 presents the results from two measurement methods regarding the teaching intervention's adaptivity. Moreover, this section examines whether the measurement methods enable statements about the construct of adaptivity.

5 Results

5.1 Analysis of the teaching interventions' criteria fulfillment

Based on the description of the five adaptivity criteria (see Sect. 2.3) and the analysis of the extent to which it is possible to make statements about these criteria using the two measurement methods used in this study (see Sect. 4.4), this and the following section focus on the results of the two studies in terms of the five adaptivity criteria. As a first step, all the teaching interventions of the four experienced math teachers from both studies are analyzed regarding the different adaptivity criteria presented in Section 2.3. This allows for identifying the advantages and disadvantages of these two methodological approaches. This assessment of the instruments was required to interpret the results of the intervention's adaptivity in Section 5.2. This selective, explorative approach enables the development of the first hypothesis regarding the model's suitability for examining the adaptivity of teaching interventions.

5.1.1 Analysis of the teaching interventions regarding the existence of barriers

Table 3 on the next page presents the total number of teaching interventions for the VLO in the left column. The right column presents the relative frequencies of teaching interventions in which there was a barrier to a student's solution process. Barriers to students' solution processes only occurred in 45–73 percent of all teaching interventions. Whether the teachers assumed a barrier in the students' solution processes could not be analyzed by VLO. However, further analysis of the interventions showed that most of the interventions without student barriers were caused by organizational issues such as lesson organization or students' processing progress (see Scharnberg, in preparation). Owing to its design, the CTI can only be used to examine cases where there are barriers to students' solution processes, resulting in barriers in 100 percent of all teaching interventions in CTI. For better comparability, further data analysis in this study only considered teaching interventions in which content-based barriers within students' solution processes occurred.

Table 3: Relative frequencies of interventions with and without barriers (VLO) (own research)

Teacher	Relative Frequency of Teaching Interventions with Barriers (C1)
Teacher A (*n* = 56)	0.45
Teacher B (*n* = 51)	0.61
Teacher C (*n* = 38)	0.47
Teacher D (*n* = 51)	0.73

5.1.2 Analysis of the teaching interventions regarding diagnosis and teachers' prior knowledge

Regarding the second criterion, which focuses on diagnoses and teachers' pre-existing knowledge, there were no results from the VLO for methodological reasons (see Sect. 4). The results of the CTI showed that all teachers had expert knowledge regarding problem-solving tasks, which was the focus of the study (see Table 4, right column). A different picture emerged among teachers regarding general and subject-specific didactical knowledge. Teacher C solved 83 percent of the tasks for general and subject-specific didactic knowledge, whereas Teacher D solved only 17 percent of the tasks. Teachers A and B solved half and two-thirds of the items, respectively.

Table 4: Relative frequencies of diagnosis and knowledge (CTI) (own research)

	CTI		
Teacher	Relative Frequency of Teaching Interventions[6] Based on Appropriate Diagnosis	Correct General and Subject-Specific Didactic Knowledge (Relative Frequencies)	Correct Expert Knowledge of Selected Problem-Solving Tasks (Relative Frequencies)[7]
Teacher A (*n* = 14)	0.21	0.50	1
Teacher B (*n* = 20)	0.25	0.67	1
Teacher C (*n* = 12)	0.75	0.83	1
Teacher D (*n* = 14)	0.50	0.17	1

These results are surprising, as the model suggests a close relationship between different knowledge areas. Regarding the diagnosis of the given cases, 21–75 percent of the teaching interventions rated as adaptive were based on appropriate diagnoses.

[6] To compare the interventions performed in the VLO (Study I) with those rated as suitable in the CTI (Study II), these are also considered to be rated as adaptive (see Sect. 4). As the number of suitable rated interventions in the CTI differs between the four teachers, the total number of teaching interventions differs between teachers (see left column).

[7] In the CTI, the four teachers answered all expert knowledge items on selected problem-solving tasks correctly.

5.1.3 Analysis of the teaching interventions regarding targeting the barriers

Regarding the third criterion, the results of the VLO showed that approximately 75 percent of all teaching interventions targeted barriers in students' solution processes, with the exception of interventions by Teacher D. These results were expected because of Teacher D's low number of interventions based on correct diagnoses (see Sect. 5.1.2). The results of the CTI show that 50–75 percent of the teaching interventions rated as adaptive targeted students' barriers in their solution processes. Furthermore, the two studies showed different results regarding the third criterion for Teachers B and D. While Teachers B and A targeted barriers with similar frequencies in both studies, Teacher D's interventions only targeted barriers in solution processes in 49 percent of the observed teaching interventions in the VLO and 71 percent of the interventions in the CTI (see Fig. 8). Consequently, each methodological approach captures different aspects of this criterion (see Sect. 6).

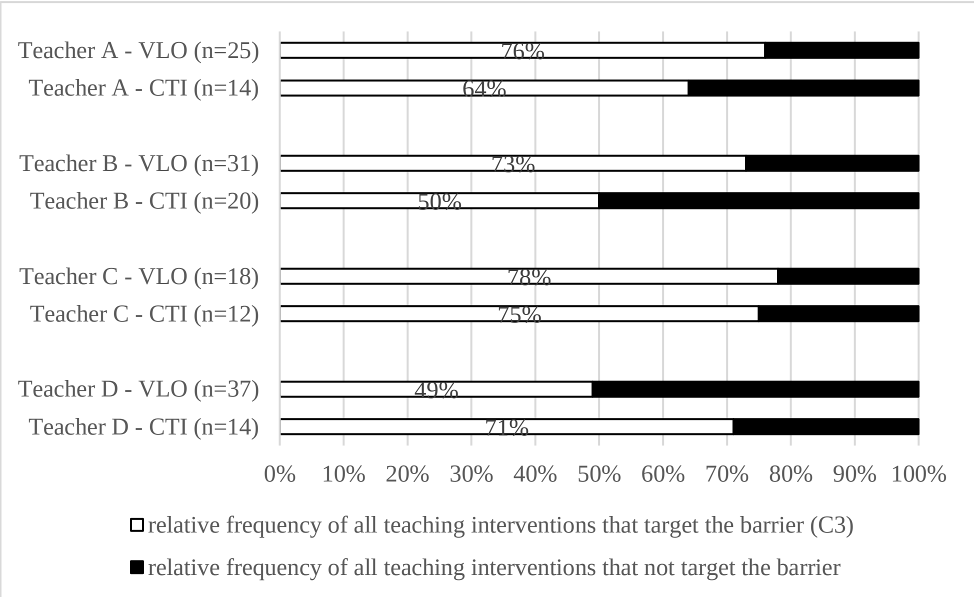

Figure 8: Relative frequencies of interventions that do and do not target barriers (VLO and CTI) (own research)

5.1.4 Analysis of the teaching interventions regarding the independent continuation of the solution process

In the VLO, 77–100 percent of all teaching interventions allowed students to continue their solution processes independently. The CTI showed similar results (see Fig. 9). In contrast to the previous criterion, Teacher D's interventions allowed students to continue their solution processes independently in the VLO, although this was true only for 86 percent of the interventions in the CTI. Teacher B's results also indicated deviations in the two studies, but the other way around: Teacher B allowed the students to independently continue their solution processes in 73 percent of the interventions in the VLO, but only in 50 percent of the interventions in the CTI. The results for Teachers A and C were similar in both studies.

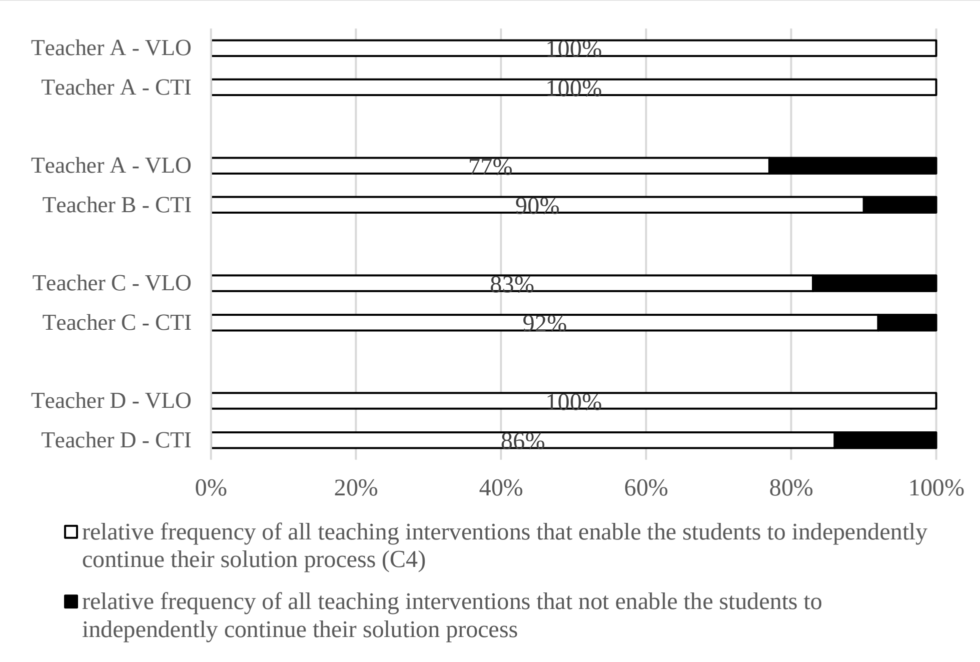

Figure 9: Relative frequencies of interventions that do and do not enable students to independently continue their solution processes (VLO and CTI) (own research)

5.1.5 Analysis of the teaching interventions regarding overcoming barriers

The results of the VLO regarding the fifth criterion demonstrate that the fulfillment of this criterion varied substantially between the teachers, ranging from 35–89 percent of all teaching interventions enabling students to overcome barriers in their solution processes (see Fig. 10). The results of the CTI showed that only half to two-thirds of all teaching interventions rated as adaptive enabled the students to overcome barriers in their solution processes. While the results for Teachers A and B were similar in both studies,[8] the different methodological approaches showed divergent results for Teachers D and C. 71 percent of Teacher D’s interventions in the CTI were suitable for enabling students to overcome barriers in their solution processes; however, only 35 percent of the interventions in the VLO actually enabled students to overcome barriers. Conversely, only 58 percent of Teacher C’s interventions enabled students to overcome barriers in the CTI, compared with 89 percent in the VLO. These divergent results may be based on the assumption that the two methodological approaches capture different aspects of this criterion (see Sect. 6).

[8] Based on the small number of interventions per teacher, smaller differences between the results of the two studies (e.g., 11 % and 13 %) could be due to random effects. Therefore, they are not considered to be relevant differences.

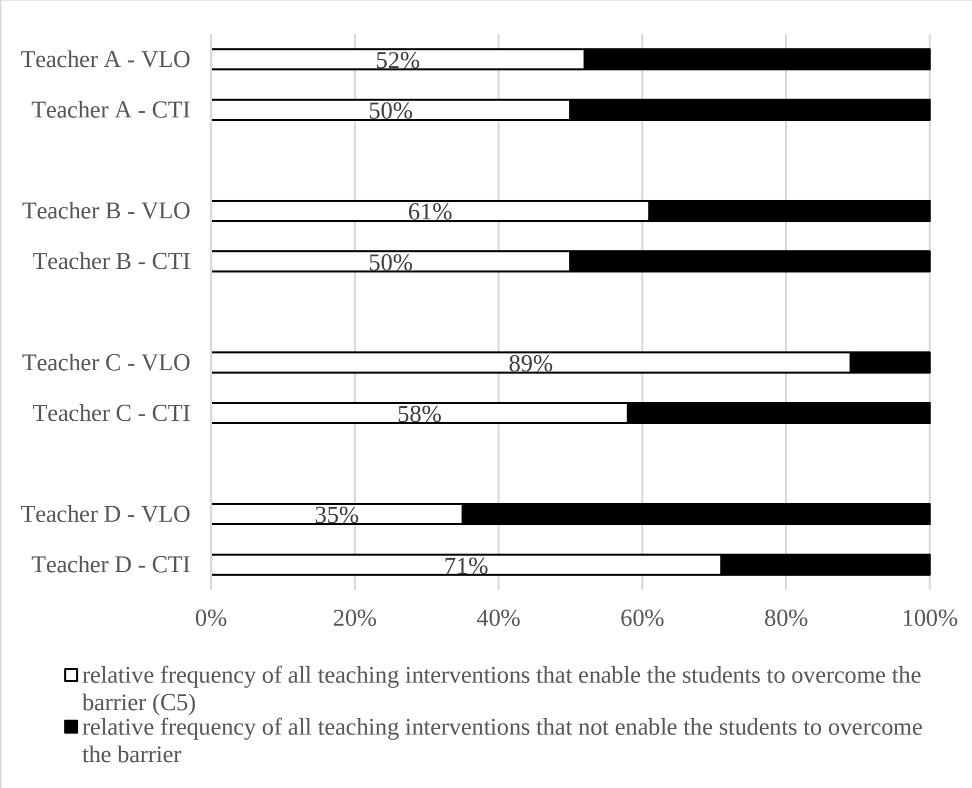

Figure 10: Relative frequencies of interventions that do and do not enable students to overcome barriers (VLO and CTI) (own research)

In summary, the results of the VLO illustrate that there were barriers to solution processes in up to three-quarters of all cases, meaning that students did not require support in at least one-quarter of cases in which teachers intervened. Moreover, the results of the CTI show that three out of four teachers in the sample made matching diagnoses in less than 50 percent of the interventions and, therefore, failed to intervene adaptively. The results of both studies demonstrated equally that 50–75 percent of the teaching interventions targeted barriers within students' solution processes. While 77–100 percent of the teaching interventions allowed the students to continue their solution processes, 11–65 percent of them did not enable them to overcome barriers in their solution processes. Consequently, placing students in charge of the learning process may occur at the expense of learning success. However, methodological approaches do not always yield the same results for all teachers. The results for Criteria 3 and 5 show larger differences between two teachers in the sample. Whether these differences are due to differences in the two methodological approaches is discussed in Section 6.

5.2 Analysis of the teaching interventions' adaptivity

Across both methodological approaches, 41 percent of Teachers B and D's interventions, 51 percent of Teacher A's interventions, and 63 percent of Teacher C's interventions could be characterized as adaptive (see Fig. 11). Considering that the teachers were chosen based on their pre-existing knowledge and teaching experience, these results were surprising. The following section presents an analysis of which adaptivity criteria the teachers struggled with the most to obtain a better understanding of possible interdependencies.

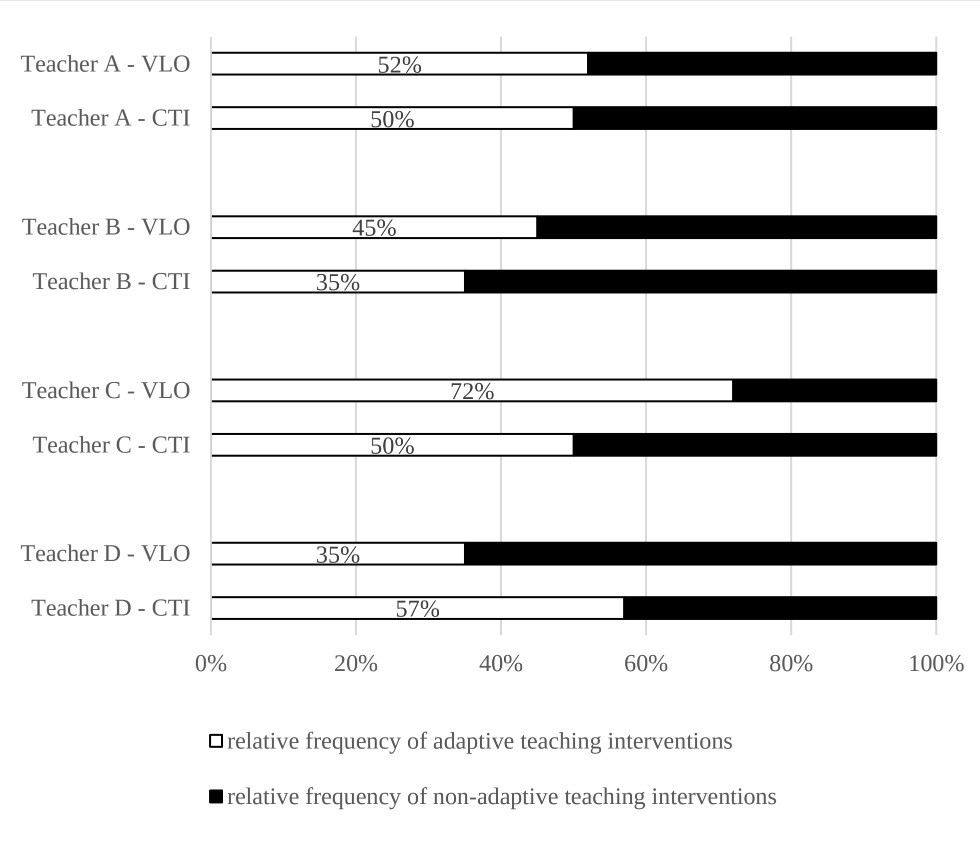

Figure 11: Relative frequencies of adaptive and non-adaptive teaching interventions (VLO and CTI) (own research)

Looking at the results of the two studies conducted separately, different measurement methods generated different results for some teachers. Whereas the VLO showed similar results regarding the relative frequency of teaching interventions rated as adaptive for Teacher A, the results for Teachers B, C, and D differed across the two studies. Possible explanations for the contrasting results from the different methodological approaches are provided in Section 6.

6 Discussion

This section highlights the conclusions from the findings concerning the adaptive teaching interventions of the four experienced math teachers according to the different measurement methods used in the two studies. Therefore, we first summarize and discuss the results presented regarding the interventions' fulfillment of the adaptivity criteria and the two methodological approaches with respect to previous research findings. Second, we address the limitations of the research and discuss implications for future research.

6.1 Discussion of the results regarding the interventions' adaptivity

The research results show that 41–63 percent of the teaching interventions of four experienced math teachers were adaptive according to the normative model presented in Section 2.3. Looking at the results from a different perspective, approximately half of the four teachers' interventions were not adaptive. The question arises as to why many of the teaching interventions were non-adaptive and which of the adaptivity criteria in the model posed the greatest hurdle for the four teachers. Based on the results, possible hurdles for teachers are discussed, following which of these results are considered in relation to the current state of research.

The results of the content-based interventions in the VLO showed that up to 55 percent of the teachers' interventions took place even though the students were not facing barriers within their learning processes. Consequently, one-quarter of all teaching interventions within the collected data were non-adaptive because, based on the model of adaptive teaching interventions, teacher intervention was not needed. To support this statement and gain a better understanding of why teachers intervene even when intervention is objectively unnecessary, the different triggers of the interventions need to be analyzed in further research. In addition, teachers should be prepared to distinguish situations in which interventions are necessary.

The CTI findings highlight that only one of the four teachers (the one with the highest general and subject-specific didactic knowledge) diagnosed solution processes and barriers correctly in 75 percent of their teaching interventions, whereas the other three teachers diagnosed them correctly in only half of their teaching interventions. These results support findings from previous research showing that teachers have difficulty diagnosing students' solution processes and barriers (Dann et al., 1999; Leiss, 2007; Meloth & Deering, 1999; Seifried & Wuttke, 2010; Tropper et al., 2015). Consequently, diagnostic competencies need to be developed and expanded for both teacher training and education at universities. Moreover, the findings on teachers' diagnoses suggest interdependencies of at least two adaptivity criteria. Because of the small sample sizes of the two studies, these interdependencies could not be analyzed. Nonetheless, further analysis is necessary to better understand these interrelations and the complexity of adaptive teaching interventions (see Sect. 6.3).

As more than three-quarters of the teaching interventions in the data collected allowed students to continue their solution processes independently, the fourth criterion does not seem to pose a major difficulty for teachers in either methodological approach.

Conversely, across teachers and measurement methods, the teachers struggled to address student barriers within one-quarter to half of their interventions. Moreover, up to 65 percent of all teaching interventions analyzed did not enable students to overcome barriers in their solution processes. Again, these results suggest the interdependencies of the different adaptivity criteria in the model, which need to be analyzed in further research with a larger sample. Regarding future teachers' sensitivity to whether students are facing barriers, teacher education needs to include training on how to properly diagnose and target a student's barriers and determine whether a student has overcome a barrier within a learning setting.

In summary, the results of these two studies represent what has been addressed in previous studies on the adaptivity of teaching interventions. First, the teachers in the sample had great difficulties in diagnosing and analyzing the solution process (see Dann et al., 1999; Leiss, 2007; Meloth & Deering, 1999; Seifried & Wuttke, 2010; Tropper et al., 2015) and determining whether a barrier in the solution process was present. Second, teachers faced difficulties in choosing adaptive interventions (see Cooper, 2009; Tropper et al., 2015), especially regarding addressing students' barriers and enabling them to overcome them. Third, the results of the study support the assumption that not all the criteria described in the normative model are built upon each other, as described in Section 2.3. While there seem to be positive interdependencies between some criteria, such as the existence and targeting of student barriers, there also seem to be negative interdependencies. The data analyses suggest that the four teachers' teaching interventions that enabled students to continue working independently were less likely to target barriers in solution processes and, especially, to overcome these barriers. Overall, these assumptions illustrate the need for research and education to focus on adaptive teaching interventions. From a macro level perspective, teacher education should focus on developing and expanding teachers' knowledge and diagnostic competencies. Simultaneously, research should focus on the interdependencies of teachers' competencies and their effects on the adaptability of interventions. From a micro level perspective, teacher training and research need to emphasize the relevance of barriers and how to overcome them, balanced by the facilitation of an autonomous learning process.

6.2 Discussion of the results regarding the two methodological approaches

The two measurement methods led to somewhat different results for the four teachers, especially regarding Criteria 3 and 5, as well as regarding the general statements that can be made about the interventions' adaptivity. This outcome suggests that the criteria may incorporate different requirements for individual teachers into each methodological approach. This difference raises the question of to what extent the two approaches measure the same or different aspects of the construct of adaptive teaching interventions and which of the two methodological approaches is more suitable for examining the adaptivity of teachers' interventions.

The VLO analysis focused on whether a teaching intervention is adaptive based on its accuracy regarding students' learning processes in terms of teachers' and students' actions in real classroom settings. By contrast, the CTI analysis focused on capturing teachers' decisions about the appropriateness of intervention prompts with respect to various constructed situations. In a real teaching situation, teachers are confronted with interventions' full complexity, for example, intervening under time constraints (Pauli & Reusser, 2006) and designing and executing interventions from scratch. The CTI focuses on thoughtfully chosen intervention options based on multiple facets of teachers' competencies, such as knowledge and diagnosis. Consequently, a far-reaching difference between the two methodological approaches may be the teacher competencies required to implement teaching interventions.

In addition to this first difference, the two methodological approaches can be understood as contrasting elements that prioritize different aspects of adaptivity when collecting and analyzing data. Whereas the VLO enabled the capture of different barriers facing students, the CTI focused more on capturing teachers' diagnoses and intervention selection from a theoretical perspective without pressure for action. The latter can be used to capture teachers' unobservable prior knowledge and their explicit diagnoses of the presented situations (Klock & Siller, 2019).

Both methodological approaches allow conclusions to be drawn about the accuracy of a chosen or implemented intervention for student barriers, as well as about the maintenance of their independence through the intervention. Moreover, conclusions regarding the suitability of an intervention for overcoming barriers can be drawn using both methodological approaches. However, both methodological approaches have strengths and weaknesses depending on the focus of the analysis. While the CTI only enables statements about potentially overcoming a barrier, the VLO provides insight into whether an intervention actually supports students in overcoming their barriers because of the possibility of analyzing students' ongoing solution processes. To create an integrated picture of the adaptivity of teaching interventions, the two measurement methods can and should be viewed as complementary, enabling comprehensive statements on the adaptivity of teaching interventions.

6.3 Study limitations and implications for future research

This study had three limitations. The first is related to the lack of diversity in the CTI items. As there were student barriers in all constructed cases and all barriers were content-related, only interventions based on existing content-based barriers were considered in the comparison of the two methodological approaches. This limitation can be addressed in future research by changing or extending the test instruments, which would allow for a greater variety of sample cases, namely those with and without different types of student barriers, to be implemented. As a result, it would also be possible to survey the first criterion more extensively using the CTI. Accordingly, the comparison between the two methodological approaches could be extended. The second limitation is related to the difference in the conditions of data collection between the two studies, as described in Section 4. Because of the different problem-solving tasks, the comparability of the two studies cannot necessarily be assured. In future studies, the same problem-solving tasks should be used to ensure better comparability. The third limitation concerns the conclusions drawn based on the chosen sample and the type of research conducted. As the results originate from an explorative study with only four teachers focusing on only task-based mathematics teaching, no causal or generalizable statements can be made. Instead, assumptions that need to be verified in further research with larger samples are developed (see Sect. 6.1 and 6.2).

Despite these limitations, the research conducted in this study represents an initial step towards analyzing the adaptivity of teaching interventions by integrating two methodological approaches. Overall, both methodological approaches, the VLO and CTI, appear suitable for analyzing and capturing the adaptivity of teaching interventions. Depending on the focus of the analysis (e.g., diagnosis and ongoing learning processes of students), both methodological approaches have strengths and weaknesses. Notwithstanding the methodological approach, the established normative model allows for a differentiated view of the adaptivity of teaching interventions, as well as of the different facets of an intervention's adaptivity. The findings of this study re-emphasize teachers' difficulties in diagnosing and targeting student barriers. Furthermore, they point to the high number of teaching interventions that take place even when students do not face barriers. The results suggest the interdependencies of various facets of the interventions, such as teachers' knowledge of a task and the adaptivity of their teaching interventions. Further research is required in this field. Additionally, different facets of adaptive teaching interventions must be integrated into teacher education to provide teachers with the best possible preparation for intervening to help students overcome barriers.

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